PRESBYTERIAN LADIES' COLLEGE
A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT
MATHEMATICAL METHODS YEAR 12 – TEST 3

DATE: 27th June 2016

Name: _Mark Allwright._

CALCULATOR FREE

Reading Time: 3 minutes

Working Time: 50 minutes

EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet (provided)

Question	Marks available	Marks awarded
1	4	
2	6	
3	6	
4	3	
5	6	
6	5	
7	6	
8	7	
9	9	
Total	**52**	

Question 1　　　　　　　　　　　　　　　　　　　　　　　　　　　　**(4 marks)**

Evaluate the following:

(a)　$\log_3 27$　　　　　　　　　　　　　　　　　　　　　　　　　　(1 mark)

$$= 3 \quad \checkmark$$

(b)　$\log_{15} 1$　　　　　　　　　　　　　　　　　　　　　　　　　(1 marks)

$$= 0 \quad \checkmark$$

(c)　$\log_{25} 0.2$　　　　　　　　　　　　　　　　　　　　　　　　(2 marks)
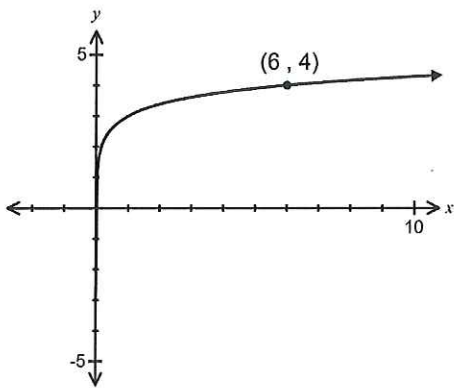
$$25^x = \frac{1}{5} \quad \checkmark$$

$$5^{2x} = 5^{-1}$$
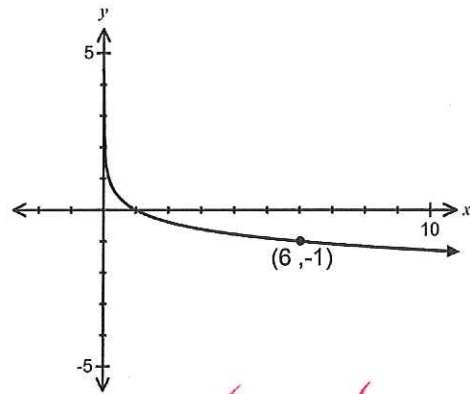
$$x = -\frac{1}{2}. \quad \checkmark$$
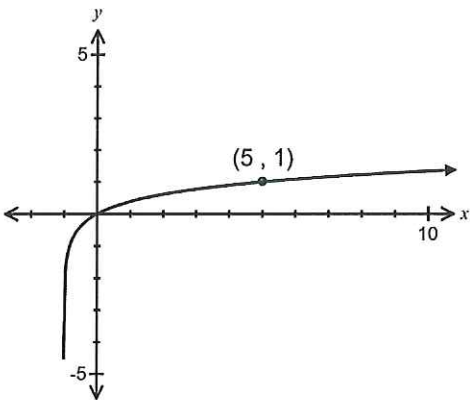
2

Question 2

(6 marks)

Match each of the following graphs with their equations from the given list.
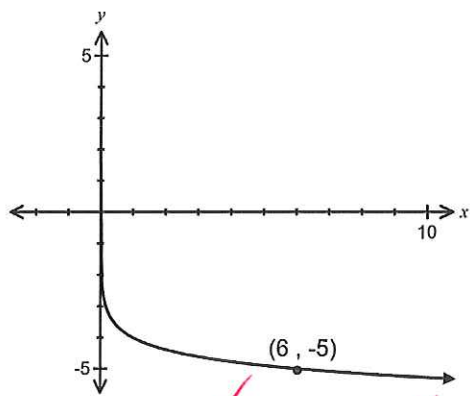A graph may have more than one matching equation. Not all equations have a matching graph.

(6 , 4)

E ✓

(6 ,-1)

D ✓ C ✓

(5 , 1)

A ✓

(6 , -5)

F ✓ I ✓

A $y = \log_6(x+1)$

B $y = \log_6(x-1)$

C $y = \log_{\frac{1}{6}}(x)$

D $y = -\log_6(x)$

E $y = \log_6(x)+3$

F $y = -\log_6(x)-4$

G $y = \log_5(x)$

H $y = \log_5(x+1)$

I $y = \log_6(\frac{1}{x})-4$

3

Question 3 (6 marks)

Differentiate with respect to x.

(a) $f(x) = \ln(2x+1)$ (1 mark)

$$f'(x) = \frac{2}{2x+1} \quad \checkmark$$

(b) $f(x) = \ln\left(\frac{x^2+2x}{x-5}\right)$ (2 marks)

$$= \ln(x^2+2x) - \ln(x-5)$$

$$f'(x) = \frac{2x+2}{x^2+2x} \checkmark - \frac{1}{x-5} \checkmark \quad \text{or} \quad \frac{x^2-10x-10}{(x^2+2x)(x-5)}$$

(c) $f(x) = \frac{2\sqrt{x}}{\ln x} = \frac{2x^{1/2}}{\ln x}$ (3 marks)

$$f'(x) = \frac{\ln x \times x^{-1/2} - \frac{1}{x} \times 2x^{1/2}}{(\ln x)^2} \quad \checkmark\checkmark$$

$$= \frac{\ln x \cdot x^{-1/2} - x^{-1/2} \times 2}{(\ln x)^2}$$

$$= \frac{\frac{1}{\sqrt{x}}(\ln x - 2)}{(\ln x)^2} \quad \checkmark = \frac{\ln x - 2}{\sqrt{x}(\ln x)^2}$$

4

Question 4   (3 marks)

Find the following indefinite integrals.
Assume denominators are greater than zero.

(a) $\int \frac{12}{x} dx$ (1 mark)

$$= 12 \ln x + C \quad \checkmark$$

(b) $\int \frac{2e^x + 2\sin x}{e^x - \cos x} dx$ (2 marks)

$$= 2 \int \frac{e^x + \sin x}{e^x - \cos x} \, dx \quad \checkmark$$

$$= 2 \ln\left(e^x - \cos x\right) + C \quad \checkmark$$

Question 5 (6 marks)

Find the exact value of x satisfying the equation

$$\left(3^x\right)\left(4^{2x+1}\right)=6^{x+2} \ .$$

Give your answer in the form $\dfrac{\ln a}{\ln b}$.

$$\ln\left(3^x \cdot 4^{2x+1}\right) = \ln\left(6^{x+2}\right) \quad \checkmark$$

$$x\ln 3 + (2x+1)\ln 4 = (x+2)\ln 6 \quad \checkmark$$

$$x\ln 3 + x\ln 4^2 + \ln 4 = x\ln 6 + \ln 6^2$$

$$x\ln 3 + x\ln 16 - x\ln 6 = \ln 36 - \ln 4 \quad \checkmark$$

$$x\left(\ln\left(\tfrac{3\times 16}{6}\right)\right) = \ln\left(\tfrac{36}{4}\right) \quad \checkmark$$

$$x\ln 8 = \ln 9$$

$$x = \frac{\ln 9}{\ln 8} \quad \checkmark$$

6

Question 6 (5 marks)

(a) The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x-2) - \ln(x^2 - 4)$.

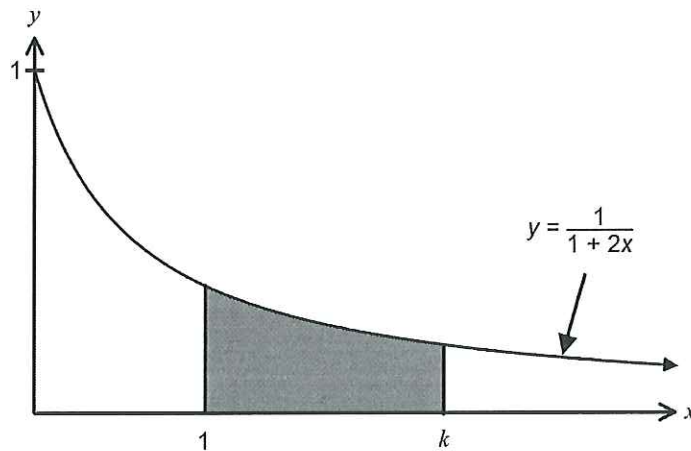Express $f(x)$ in the form $\ln\left(\dfrac{x}{x+a}\right)$. (2 marks)

$$f(x) = \ln\left(\frac{x(x-2)}{x^2-4}\right) \checkmark$$

$$= \ln\left(\frac{x(x-2)}{(x-2)(x+2)}\right)$$

$$= \ln\left(\frac{x}{x+2}\right) \checkmark$$

(b) Evaluate $\log\dfrac{1}{2} + \log\dfrac{2}{3} + \log\dfrac{3}{4} + \log\dfrac{4}{5} + \ldots + \log\dfrac{8}{9} + \log\dfrac{9}{10}$. (3 marks)

$$\log\left(\frac{1 \times 2 \times 3 \times 4 \times \ldots \times 8 \times 9}{1 \times 2 \times 3 \times 4 \times \ldots \times 9 \times 10}\right) \checkmark$$

$$= \log\left(\frac{1}{10}\right) \checkmark$$

$$= -\log 10$$

$$= -1 \checkmark$$

Question 7

(6 marks)

The shaded area is 0.2 units^2.

Find k exactly.

$$\int_{1}^{K} \frac{1}{1+2x} \, dx = 0.2 \quad \checkmark$$

$$\left[\frac{1}{2} \ln(1+2x) \right]_{1}^{k} = 0.2 \quad \checkmark\checkmark$$

$$\frac{1}{2} \left[\ln(1+2k) - \ln(1+2) \right] = 0.2 \quad \checkmark$$

$$\ln\left(\frac{1+2k}{3} \right) = 0.4 \quad$$

$$e^{0.4} = \frac{1+2k}{3} \quad \checkmark$$

$$1+2k = 3e^{0.4}$$

$$2k = 3e^{0.4} - 1$$

$$K = \frac{3e^{0.4} - 1}{2} \quad \checkmark$$

8

Question 8 (7 marks)

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable T, where $3 \leq t \leq 14$ minutes.

(a) Sketch the probability density function $f(t)$ for the interval between each firing on the axes below.

(2 marks)
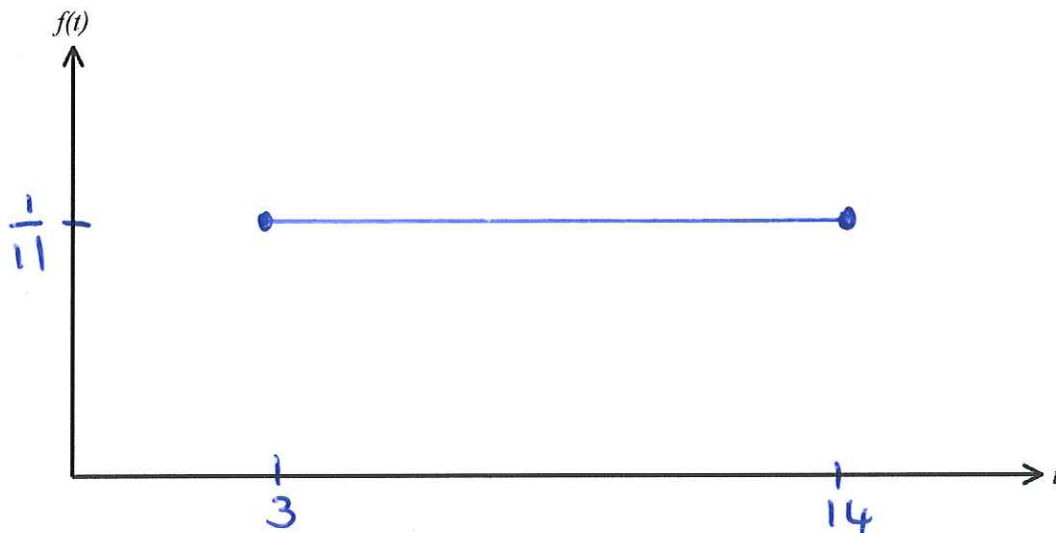
(b) Determine the probability that a randomly chosen interval between firings is

(i) at least seven minutes. (1 mark)

$$P(7 \leq x \leq 14) = \frac{7}{11} \checkmark$$

(ii) at least six minutes given that it is less than ten minutes. (2 marks)

$$P(x \geq 6 \mid x \leq 10) = \frac{4/11}{7/11} \checkmark = \frac{4}{7} \checkmark$$

(c) Determine the value of t for which $P(T < t) = P(T > 4t)$ (2 marks)

$$\frac{t-3}{11} = \frac{14-4t}{11} \checkmark$$

$$5t = 17$$

$$t = \frac{17}{5} = 3\frac{2}{5} \checkmark$$

9

Question 9 (9 marks)

The continuous random variable X is define by the probability density function

$$f(x) = \begin{cases} \dfrac{2x}{9} & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine $E(X)$. (2 marks)

$$E(x) = \int_0^3 \frac{2x^2}{9}\, dx \quad \checkmark$$

$$= \left[\frac{2x^3}{27}\right]_0^3 = 2 \quad \checkmark$$

(b) The Variance of X, $\text{Var}(X)$, is $\dfrac{1}{2}$.

(i) Determine $E(4X+3)$ (1 mark)

$$= 4 \times 2 + 3$$
$$= 11 \quad \checkmark$$

(ii) Determine $\text{Var}(4X+3)$ (1 mark)

$$\frac{1}{2} \times 4^2 = 8 \quad \checkmark$$

Question 9   continued   **(9 marks)**

(c)   Determine the cumulative distribution function $F(x)$ (3 marks)

$$\int_0^K \frac{2x}{9} \, dx \quad \cdot \quad \checkmark$$

$$= \left[\frac{x^2}{9} \right]_0^K$$

$$= \frac{K^2}{9} \quad \checkmark$$

$$P(X \le x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \frac{x^2}{9} & \text{for} \quad 0 \le x \le 3 \\ 1 & \text{for} \quad x > 3 \end{cases} \quad \checkmark$$

(d)   Calculate $P(1 < x < 2)$ (2 marks)

$$P(1 < x < 2) = \frac{2^2}{9} - \frac{1^2}{9} \quad \checkmark$$

$$= \frac{3}{9}$$

$$= \frac{1}{3} \quad \checkmark$$

End of Test

Additional page for working.